Representation Zeta Functions of Norm 1 Subgroups of a Local Division Algebra

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Let D be a central division algebra of dimension ℓ^2 over a non-archemedian local field K. Let $\mathcal{O}$ denote the maximal compact subring and $\mathcal{P}$ the maximal ideal of D.

In his 1981 paper, H. Koch investigated the notion of conjugation by invertible elements of $\mathcal{O}$ modulo powers of $\mathcal{P}$.

We focus our attention on the group $\mathrm{SL}_1(\mathcal{O})$ of elements of reduced-norm 1 in D and its first congruence subgroup $\mathrm{SL}_1^1(\mathcal{O}) = \mathrm{SL}_1(\mathcal{O}) \cap (1+\mathcal{P})$. Applying some of the methods described in Koch, we compute the orbits of the action of $\mathrm{SL}_1^1(\mathcal{O})$ on quotients of the Lie-lattice of elements of reduced trace 0 in $\mathcal{P}$ modulo powers of $\mathcal{P}$, in the case where ℓ is a prime number distinct from the residual characteristic of K.

Under some additional assumption on the field K, we will present a connection between the character theory of $\mathrm{SL}_1^1(\mathcal{O})$ and the aforementioned orbits, via the Kirrilov orbit method for a specific class p-adic groups. In particular, by applying a p-adic formalism developed by Avni, Klopsch, Onn and Voll for such groups, we will obtain the representation zeta function of $\mathrm{SL}_1^1(\mathcal{O})$.

Time permitting, we will discuss the connection to the representation zeta function of $\mathrm{SL}_1(\mathcal{O})$.