The Congruence Subgroup Problem for Automorphism Groups

November 29, 2016

Abstract

The classical congruence subgroup problem asks whether every finite quotient of $G = GL_n(\mathbb{Z})$ comes from a finite quotient of $\mathbb{Z}$. I.e. whether every finite index subgroup of G contains a principal congruence subgroup of the form $G(m) = \ker(G \to GL_n(\mathbb{Z}/m\mathbb{Z}))$ for some $m \in \mathbb{N}$. If the answer is affirmative we say that G has the congruence subgroup property (CSP). It was already known in the $19^{\underline{th}}$ century that $GL_2(\mathbb{Z})$ has many finite quotients which do not come from congruence considerations. Quite surprising, it was proved in 1962 that for $n \geq 3$, $GL_n(\mathbb{Z})$ does have the CSP.

Observing that $GL_n(\mathbb{Z}) \cong Aut(\mathbb{Z}^n)$, one can generalize the congruence subgroup problem as follows: Let Δ be a group. Does every finite index subgroup of $G = Aut(\Delta)$ contain a principal congruence subgroup of the form $G(M) = \ker(G \to Aut(\Delta/M))$, for some finite index characteristic subgroup $M \leq \Delta$? Very few results are known when Δ is not abelian. For example, we do not know if $Aut(F_n)$ for $n \geq 3$ has the CSP. However, in 2001 Asada proved, using tools of algebraic geometry, that $Aut(F_2)$ does have the CSP.

On the talk, we will discuss some new results considering the above problem, using some basic methods of profinite groups.