Modular Galois representations

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Let p be a prime and let f be a modular form. Let ρ_f be the two dimensional p-adic Galois representation attached to f. We are interested in the (local) behaviour of ρ_f when f is a p-ordinary form of weight at least 2. A result of Mazur-Wiles says that when f is p-ordinary, ρ_f restricted to the decomposition group G_p at p is reducible. Greenberg asked the natural question; when does it split? It is not too hard to see that $\rho_f|_{G_p}$ splits if f has complex multiplication (CM). In this talk, we will discuss the converse, i.e.,

$$\rho_f|_{G_p} \text{ splits} \stackrel{?}{\Longrightarrow} f \text{ has CM.}$$

We use deformation theory of Galois representations and the theory of p-adic families of modular forms (Hida families) to generate various non-trivial examples in support of the converse. I will describe these ideas in sufficient detail to suit a general audience.