Holomorphic extension of fundamental solutions of elliptic linear partial differential operators with analytic coefficients

Abstract

We start with a simple fact: the fundamental solutions of the Laplacian in $\mathbb{R}^n$ can be continued as multi-valued analytic functions in $\mathbb{C}^n$ up to the complex bicharacteristic conoid. This extension ramifies around the complex isotropic cone: $z_1^2 + \cdots z_n^2 = 0$ and has "moderate growth".

For an elliptic linear partial differential operator of the second order with analytic coefficients and simple complex characteristics in an open set $\Omega \subset \mathbb{R}^n$, this may be generalized: every fundamental solution can be continued at least locally as a multi-valued analytic function in $\mathbb{C}^n$ up to the complex bicharacteristic conoid. This holomorphic extension is ramified around the bicharacteristic conoid and belongs to the so-called Nilsson class ("moderate growth").

In fact, those results remain true for such operators with degree bigger than 4 , but the proofs are different due to the lack of natural geodesic distance associated to the operators.

Those results may be connected with D-module theory, and more precisely with regular holonomic D-Modules. We'll explain this link and state a general conjecture.