The weight, density and Lindelöf number in spaces and topological groups

Mikhail G. Tkachenko
Metropolitan Autonomous University, Mexico City

It is well known that every *regular* space X satisfies $w(X) \leq 2^{d(X)}$, where $w(X)$ and $d(X)$ are the weight and density of X, respectively. For Tychonoff spaces, W. Comfort and A. Hager established a sharper result. They proved that the number of continuous real-valued functions on a Tychonoff space X, say $C(X)$, satisfies $w(X) \leq |C(X)| \leq w(X)^{wl(X)}$, where $wl(X)$ is the *weak Lindelöf number* of X. Notice that $w(X)^{wl(X)} \leq 2^{d(X)}$ for every *regular* space X.

We present new upper bounds for the weight of spaces and topological groups which contain a dense Lindelöf Σ-subspace. One of our principal results states that if Y is a dense subspace of a Tychonoff space X, then $w(X) \leq |C(X)| \leq nw(Y)^{Nag(Y)}$, where $Nag(Y)$ is the *Nagami* number of Y. In particular, if a regular Lindelöf Σ-space X satisfies $nw(X) = \kappa^\omega$ for some $\kappa \geq \omega$, then $w(X) = nw(X)$ and $w(X) = w(\beta X) = |C(X)| = \kappa^\omega$. Therefore, the cardinality of $C(X)$ is completely defined by the weight of X in this case.

The upper bounds for the weight of topological groups are even better. We show that if a Lindelöf Σ-group G is a dense subgroup of a topological group H, then $w(H) = w(G) \leq \psi(G)^\omega$. Similarly, if a Lindelöf Σ-space X generates a dense subgroup of a topological group H, then $w(H) \leq 2^{\psi(X)}$.