ON THE FREE SET NUMBER OF TOPOLOGICAL SPACES AND THEIR G_δ-MODIFICATIONS

ISTVÁN JUHÁSZ

A transfinite sequence of points of a topological space X is a *free sequence* in X if the closure of any initial segment of it is disjoint from the closure of the corresponding final segment. A *subset* $S \subset X$ is *free* in X if it admits a well-ordering that turns it into a free sequence in X. We let $\mathrm{F}(X) = \sup\{|S| : S \text{ is free in } X\}$, and call it the *free set number* of X.

We present several new inequalities involving $\mathrm{F}(X)$ and $\mathrm{F}(X_\delta)$, where X_δ is the G_δ-modification of X:

- $\mathrm{L}(X) \le 2^{2^{\mathrm{F}(X)}}$ if X is T_2 and $\mathrm{L}(X) \le 2^{\mathrm{F}(X)}$ if X is T_3;
- $|X| \le 2^{2^{\mathrm{F}(X)\cdot\psi_c(X)}} \le 2^{2^{\mathrm{F}(X)\cdot\chi(X)}}$ for any T_2-space X;
- $\mathrm{F}(X_\delta) \le 2^{2^{2^{\mathrm{F}(X)}}}$ if X is T_2 and $\mathrm{F}(X_\delta) \le 2^{2^{\mathrm{F}(X)}}$ if X is T_3.

We also present several forcing constructions of spaces that shed some light on the sharpness of these inequalities.

All the results are joint with L. Soukup and Z. Szentmiklóssy

Alfréd Rényi Institute of Mathematics
E-mail address: juhasz@renyi.hu